Global Hecke Algebras

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$G_v = GL_2(k_v)$

$G = GL_2 \quad K_v = GL_2(\mathcal{O}_v)$

$$\mathcal{H}_v = \begin{cases} \{ f : G_v \to \mathbb{C} \mid \text{locally constant, compact support} \}, & \text{non-arch.} \\ U(\mathfrak{g}), & \text{complex} \\ U(\mathfrak{g}) \oplus U(\mathfrak{g}) * \varepsilon_-, & \text{real} \end{cases}$$

ε_- = dirac measure at $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ in G_v.

$\mathcal{H}(\mathbb{A}) = \bigotimes' \mathcal{H}_v$ r.t.p w.r.t. $\mathbb{1}_{K_v}$

Let ρ be an admissible irreducible unitary representation of $G(\mathbb{A})$ on $\mathcal{H} = \bigotimes_v B_v$

$\subset L^2_0(G(k) \backslash G(\mathbb{A}), \omega)$

ω = unitary character of $Z(k) \backslash Z(\mathbb{A})$

Let θ be an irred. repn. of K. Then $\theta = \bigotimes \theta_v$ with θ_v irred. repn. of K_v, $\theta_v = \text{id}$ for almost all v

$B_v(\theta_v)$ = the isotypic component of $\rho_v|_{K_v}$ corresponding to θ_v

$\forall v$, ρ defines an irred. admiss. repn ρ_v^f of $\mathcal{H}_v$ on B_v^f = K_v-finite vectors in B_v.

$$\rho_v^f(F)g = \int F(x) \rho_v(x) g \, dx$$

We have $B_v^f = \bigoplus_{\theta_v \in \hat{K}_v} B_v(\theta_v)$

$\hat{K}_v$ = set of equiv. classes of irreducible representation of K_v

We get an irreducible representation $\pi = \otimes \rho_v^f$ of $\mathcal{H}(\mathbb{A})$ on $V = \otimes B_v^f$

- $V \simeq B^f$
- $\forall\ \varphi \in B^f \subset L_0^2(G(\mathbb{R})\backslash G(\mathbb{A}))$, we have $\varphi \in C^\infty(G(\mathbb{A}))$ and rapidly decreasing on the siegel domain.
- π is given by $\pi(\mu)\varphi = \varphi * \check{\mu}$.

$\pi = \otimes \rho_v^f$

$\mu \in \mathcal{H}(\mathbb{A})$, $\varphi \in V \simeq B^f$

$\check{\mu}(x) = \mu(x^{-1})$

<u>Conclusion</u> Given topologically irreducible subrepresentation ρ of the unitary repn of $G(\mathbb{A})$ on $L_0^2(G(\mathbb{R})\backslash G(\mathbb{A}), \omega)$ we get an algebraically irreducible representation π of $\mathcal{H}(\mathbb{A})$ on a space of cusp forms of $G(\mathbb{R})\backslash G(\mathbb{A})$.